



Answer all the questions

1. For the function $f(x) = x^3 - 6x^2 + 2$ find, [10]
- (i) It's critical, and inflection points.
 - (ii) The intervals on which $f(x)$ is increasing and decreasing.
 - (iii) The intervals on which $f(x)$ is concave up and down.
 - (iv) It's relative maximum and minimum by using 1st and 2nd derivative test.
 - (v) It's x -intercept and y -intercepts. Finally draw the graph of $f(x)$ using the above in formations.
2. (a) For the given function $f(x, y) = \cos(xy^2 - 4x)$, [6]
- (i) Find the slope of the surface $z = f(x, y)$ in the x and y direction at the point $(2, 1)$
 - (ii) Verify mixed second order partial derivatives are same or not?
- (b) Let f be a differentiable function of one variable, and let [2]
- $$w = f(\ln xy - x^2 - y^2), \text{ Find } \frac{\partial w}{\partial x} - \frac{\partial w}{\partial y}.$$
- (c) Using chain rule find $\frac{\partial w}{\partial \theta}$, where [2]
- $$w = 4x^3 + 4y^3 + z^3, \quad x = \rho \cos \theta, \quad y = \sin \theta \cos \theta, \quad z = \rho \sin \theta \sin \theta.$$
- Also show the tree diagram.
3. (a) Find the solution of the given differential equations [8]
- (i) $t^3 y' + 3t^2 y = t^3 - t + 1, \quad y(1) = 1$
 - (ii) $y' = \frac{y \cos x}{1+2y^3}, \quad y(0) = 1$
- (b) Determine the values of r for which the given differential equation has solutions of the form t^r : [2]
- $$t^2 y'' + 8t y' + 12y = 0$$