



United International University

School of Science and Engineering

Final Examination Trimester Fall 2018

Course Title: Linear Algebra, Ordinary & Partial Differential Equations

Course Code: Math 153 Marks: 40 Time: 2 Hours

There are 5 questions. Answer any 4 of them. Answer all parts of a question together.

1. a) Solve: $2 \frac{d^2y}{dt^2} - 5 \frac{dy}{dt} - 3y = 0$; $y(0) = 0$, $y'(0) = -1$. [3]
- b) The population of a metropolitan city grows at a rate proportional to the population present at time t . The initial population of P_0 increases by 23% in 15 years. What will be the population in 35 years? How long will it take to 4 times of the initial population? [5]
- c) Solve: $(D^4 + 6D^2 + 9)y = 0$. [2]
2. a) Solve the following differential equations: [7]
- i. $(x - y^3 + y^2 \ln x)dx = (3xy^2 + 2y \cos x)dy$
- ii. $x \frac{dy}{dx} = y + \sqrt{x^2 - y^2}$; $x > 0$
- b) Solve: $y'' + 4y' + 13y = 3 \cos 2x \sin 2x$. [3]
3. a) Find the series solution in powers of x about $x_0 = 0$ for the differential equation $(1 - x)y'' + xy' - y = 0$. [5]
- b) Find the general solution of the system of differential equations $X' = \begin{pmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix} X$ [5] and describe the behavior of the solution as $t \rightarrow \infty$.
4. a) Use the method of reduction of order to find a second solution of the differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0$, $t > 0$; $y_1(t) = t$. [5]
- b) Solve: $(D^3 - 2D + 4)y = 5t^2 - e^t \cos^2 t$. [5]
5. a) A tank contains 35 pounds of salt dissolved in 175 gallons of water. A water containing 4 lb of salt/gal is entering the tank at a rate of 5 gal/min and the well-stirred mixture is draining from the tank at the same rate. Find the amount of salt in the tank after a long time. [5]
- b) Solve: $(D^2 - 2D + 3)y = e^{-t} \sinh 2x - 7 \sin 3x \sin 2x$. [5]